

# Exploring Symmetry Breaking at the Dicke Quantum Phase Transition

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We study symmetry breaking at the Dicke quantum phase transition by coupling a motional degree of freedom of a Bose-Einstein condensate to the field of an optical cavity. Using an optical heterodyne detection scheme we observe symmetry breaking in real-time and distinguish the two superradiant phases. We explore the process of symmetry breaking in the presence of a small symmetry-breaking field, and study its dependence on the rate at which the critical point is crossed. Coherent switching between the two ordered phases is demonstrated.

PACS numbers: 37.30.+i, 42.50.-p, 05.30.Rt, 11.30.Qc

Spontaneous symmetry breaking at a phase transition is a fundamental concept in physics [1]. At zero temperature, it is caused by the appearance of two or more degenerate ground states in the Hamiltonian. As a result of fluctuations, a macroscopic system evolves into one particular ground state which does not possess the same symmetry as the Hamiltonian. Finding a clean testing ground to experimentally study the process of symmetry breaking is notoriously difficult as external fluctuations and asymmetries have to be minimized or controlled. The protected environment of atomic quantum gas experiments and the increasing control over these systems offer new prospects to experimentally approach the concept of symmetry breaking. Recently, rapid quenches across a phase transition were studied in multi-component Bose-Einstein condensates [2–4] and optical lattices [5, 6]. Such a non-adiabatic quench causes a response of the system at correspondingly high energies. Therefore, a central characteristic of a phase transition, which is its diverging susceptibility to perturbations, remains partially hidden.

In this work we study the symmetry breaking process while slowly varying a control parameter several times across a zero-temperature phase transition. Compared to quenching, this allows us to explore the low energy spectrum of the system which probes its symmetry most sensitively. For very slow crossing speeds we identify the presence of a residual symmetry breaking field of varying strength. Larger values of this residual field can be correlated to the repeated observation of one particularly ordered state. For increasingly steeper ramps across the phase transition the influence of the symmetry breaking field almost vanishes.

We investigate the symmetry breaking in the motional degree of freedom of a Bose-Einstein condensate coupled to a single mode of an optical cavity. Our system realizes the Dicke model [7–9] which exhibits a second-order zero-temperature phase transition [10–13]. The broken symmetry is associated with the formation of one of two identical atomic density waves, which are shifted by half an optical wavelength [8, 9, 14, 15]. Using an interferometric heterodyne technique, we monitor the symmetry-

breaking process in real time while crossing the transition point. A similar technique has been used to test self-organization in a classical ensemble of laser-cooled atoms [15], where the symmetric phase is stabilized by thermal energy rather than kinetic energy [16].

The Dicke model [7] considers the interaction between  $N$  two-level atoms and the quantized field of a single-mode cavity, which is described by the Hamiltonian

$$\hat{H} = \hbar\omega_0\hat{J}_z + \hbar\omega\hat{a}^\dagger\hat{a} + \frac{2\hbar\lambda}{\sqrt{N}}(\hat{a}^\dagger + \hat{a})\hat{J}_x. \quad (1)$$

Here,  $\hat{a}$  and  $\hat{a}^\dagger$  denote the annihilation and creation operators for the cavity mode at frequency  $\omega$ , and  $\hat{\mathbf{J}} =$

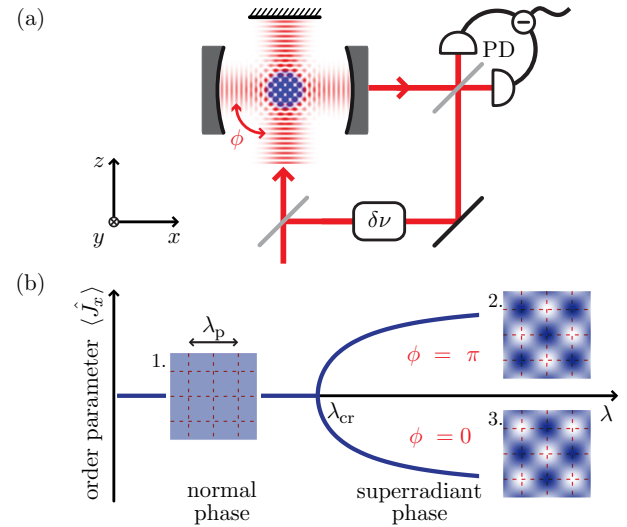


FIG. 1. (Color online) (a) Experimental setup. A Bose-Einstein condensate is placed inside an optical cavity and driven by a far-detuned standing-wave laser field (wavelength  $\lambda_p$ ) along the  $z$ -axis. Phase and amplitude of the intracavity field are measured with a balanced heterodyne setup (PD: photodiodes). (b) Steady-state order parameter  $\langle \hat{J}_x \rangle$  as a function of coupling strength  $\lambda$ , with corresponding atomic density distributions (1.-3.). The order parameter vanishes in the normal phase (1.) and bifurcates at the critical point  $\lambda_{cr}$ , where a discrete  $\lambda_p/2$ -spatial symmetry is broken. The two emergent superradiant phases (2. and 3.) can be distinguished via the relative time-phase  $\phi$ .

$(\hat{J}_x, \hat{J}_y, \hat{J}_z)$  describes the atomic ensemble with transition frequency  $\omega_0$  in terms of a pseudospin of length  $N/2$ . The cavity light field couples with coupling strength  $\lambda$  to the collective atomic dipole  $\hat{J}_x$ . In the thermodynamic limit, the Dicke model exhibits a zero-temperature phase transition from a normal to a superradiant phase when the control parameter  $\lambda$  exceeds a critical value given by  $\lambda_{\text{cr}} = \sqrt{\omega\omega_0}/2$  [10–12]. Simultaneously, the parity symmetry of the Dicke Hamiltonian, given by the invariance under the transformation  $(\hat{a}, \hat{J}_x) \rightarrow (-\hat{a}, -\hat{J}_x)$ , is spontaneously broken [13]. While parity is conserved in the normal phase with  $\langle \hat{a} \rangle = 0 = \langle \hat{J}_x \rangle$ , two equivalent superradiant phases (denoted by even and odd) emerge for  $\lambda > \lambda_{\text{cr}}$ , which are characterized by  $\langle \hat{J}_x \rangle \leq 0$  and  $\langle \hat{a} \rangle \geq 0$ , respectively (Fig. 1b).

In our experiment [8] we couple motional degrees of freedom of a Bose-Einstein condensate (BEC) with a single cavity mode using a transverse coupling laser (Fig. 1a). Within a two-mode momentum expansion of the matter-wave field, the Hamiltonian dynamics of this system is described by the Dicke model (Eq. 1) [8, 9, 17] where the effective atomic transition frequency is given by  $\omega_0 = 2\omega_r$  with the recoil frequency  $\omega_r = \hbar k^2/2m$ , the atomic mass  $m$  and the wavelength  $\lambda_p = 2\pi/k$  of the coupling laser. The frequency and power of this laser controls the effective mode frequency  $\omega$  and the coupling strength  $\lambda$ , respectively [8]. Above a critical laser power, the discrete  $\lambda_p/2$ -spatial symmetry, defined by the optical mode structure  $u(x, z) = \cos(kx)\cos(kz)$ , is spontaneously broken and the condensate exhibits either of two density waves (Fig. 1b). Correspondingly, the atomic order parameter  $\langle \hat{J}_x \rangle$ , given by the population difference between the even ( $u(x, z) > 0$ ) and odd ( $u(x, z) < 0$ ) sublattice, exhibits a negative or positive macroscopic value, while the emergent coherent cavity field oscillates (for  $\omega \gg \kappa$ ) either in ( $\phi = 0$ ) or out of phase ( $\phi = \pi$ ) with the coupling laser.

As described previously [8], we prepare BECs of typically  $2 \times 10^5$   $^{87}\text{Rb}$  atoms in a crossed-beam dipole trap centered inside an ultrahigh-finesse optical Fabry-Perot cavity, which has a length of  $176\text{ }\mu\text{m}$ . The transverse coupling laser at wavelength  $\lambda_p = 784.5\text{ nm}$  is red-detuned by typically ten cavity linewidths  $2\kappa = 2\pi \times 2.5\text{ MHz}$  from a  $\text{TEM}_{00}$  cavity mode, realizing the dispersive regime  $\omega \gg \omega_0$  of the Dicke model. We monitor amplitude and phase of the intracavity field in real-time using a balanced heterodyne detection scheme (Fig. 1a). Due to slow residual drifts of the differential path length of our heterodyne setup, which translate into drifts of the detected phase signal of about  $0.1\pi/\text{s}$ , we cannot relate the phase signals between consecutive experimental runs separated by 60 s.

To observe symmetry breaking, we gradually increase the coupling laser power across the critical point (Fig. 2a). The transition from the normal to the superradiant phase is marked by a sharp increase of the mean in-

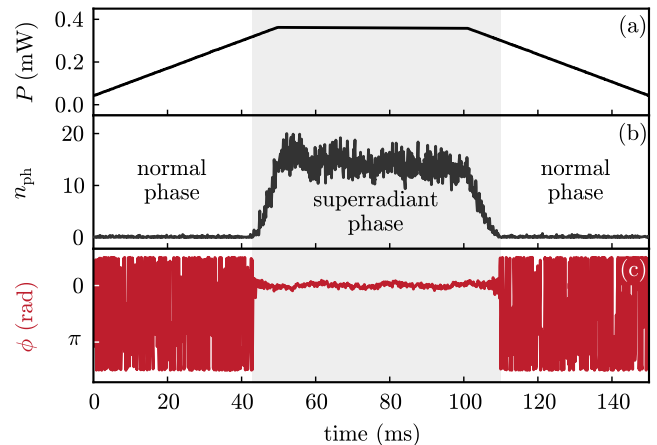


FIG. 2. (Color online) Observation of symmetry breaking and steady-state superradiance. Shown are simultaneous traces of (a) the coupling laser power  $P$ , (b) the mean intracavity photon number  $n_{\text{ph}}$ , and (c) the relative time-phase  $\phi$  between coupling laser and cavity field (both averaged over  $150\text{ }\mu\text{s}$ ). The coupling laser frequency is red-detuned by  $31.3(2)\text{ MHz}$  from the empty cavity resonance and the atom number is  $2.3(5) \times 10^5$ . Residual atom loss causes a slight decrease of the cavity photon number in the superradiant phase.

tracavity photon number (Fig. 2b). Simultaneously, the time-phase  $\phi$  between the two light fields locks to a constant value, implying that the symmetry of the system has been broken (Fig. 2c). The observation of a constant time-phase above threshold confirms that the system reaches a steady-state superradiant phase in which the induced cavity field oscillates at the coupling laser frequency. When lowering the laser power to zero again, the system recovers its initial symmetry and a pure BEC is retrieved, as was inferred from absorption imaging after free ballistic expansion.

To identify the two different superradiant states (Fig. 1b), we cross the phase transition multiple times within one experimental run (Fig. 3a). Above threshold, the corresponding phase signal takes always one of two constant values. From multiple traces of this type we extract a time-phase difference of  $1.00(2) \times \pi$  between the two superradiant phases, where the statistical error can be attributed to residual phase drifts of our detection system.

If the system was perfectly symmetric, the two ordered phases would be realized with equal probabilities, when repeatedly crossing the phase transition. However, the presence of any symmetry-breaking field will always drive the system into the same particularly ordered state when adiabatically crossing the critical point. We experimentally quantify the even-odd imbalance by performing 156 experimental runs (similar to Fig. 3a), in each of which the system enters the superradiant phase ten times within 1 s. A measure for the even-odd imbalance is given by the parameter  $\epsilon = (m_1 - m_2)/10$ , where  $m_2 \leq m_1$  de-

note the number of occurrences of the two superradiant configurations in individual traces. In 73 % of the traces, the system realized ten times the same time-phase, corresponding to the maximum imbalance of  $\epsilon = 1$  (Fig. 3b). However, 12 % of the runs exhibited an imbalance below 0.5, which is not compatible with a constant even-odd asymmetry.

We attribute our observations to the finite spatial extension of the atomic cloud. This can result, even for zero coupling  $\lambda$ , in a small, but finite population difference between the even and odd sublattice, determined by the spatial overlap  $\mathcal{O}$  between the atomic column density  $n(x, z)$  (normalized to  $N$ ) and the optical mode profile  $u(x, z)$ . This asymmetry enters the two-mode description (Eq. 1) via the symmetry-breaking term  $2\hbar\lambda\mathcal{O}(\hat{a}^\dagger + \hat{a})/\sqrt{N}$ , and renormalizes the order parameter  $\langle \hat{J}_x \rangle$  by the additive constant  $\mathcal{O}$ . The resulting coherent cavity field below threshold drives the system dominantly into either of the two superradiant phases, depending on the sign of  $\mathcal{O}$ . In the experiment, the resulting even-odd imbalance is likely to change between experimental runs, as the overlap integral  $\mathcal{O}$  depends  $\lambda_p$ -periodically on the relative position between the mode structure  $u(x, z)$  and the center of the trapped atomic cloud, with amplitude  $\mathcal{O}_0$ . We can exclude a drift of the relative trap position by more than half a wavelength  $\lambda_p$  on the timescale given by our probing time of 1 s, as it would lead to equal probabilities of the two phases, precluding spontaneous symmetry breaking.

The openness of the system provides us with direct experimental access to the symmetry-breaking field proportional to  $\mathcal{O}$ . Indeed, we detect a small coherent cavity field ( $n_{\text{ph}} < 0.02$ ) in the normal phase whose magnitude varies between experimental runs. In all runs exhibiting an imbalance of  $\epsilon = 1$  (Fig. 3b), the relative time-phases of the cavity field detected below and above threshold are equal. Furthermore, the even-odd imbalance increases significantly with the light level observed below threshold. Post-selection of those 10 % of the runs with the smallest light level yields a much smaller mean imbalance (Fig. 3b, inset).

In general, the influence of a symmetry breaking field becomes negligible, if the mean value of the order parameter, induced by this field, is smaller than the quantum or thermal fluctuations present in the system. From a mean-field calculation performed in the Thomas-Fermi limit for  $N = 2 \times 10^5$  harmonically trapped atoms, we estimate a maximum order parameter of  $\mathcal{O}_0 = 40$  for zero coupling strength, corresponding to an even-odd population difference of 40 atoms. This value is much smaller than the uncertainty  $\Delta J_x = \sqrt{N}/2 = 224$ , given by vacuum fluctuations of the excited momentum mode. Therefore, one expects in the extreme case of a sudden quench of the coupling strength beyond  $\lambda_{\text{cr}}$ , that the apparent symmetry is spontaneously broken, resulting in nearly equal probabilities of the two superradiant phases.

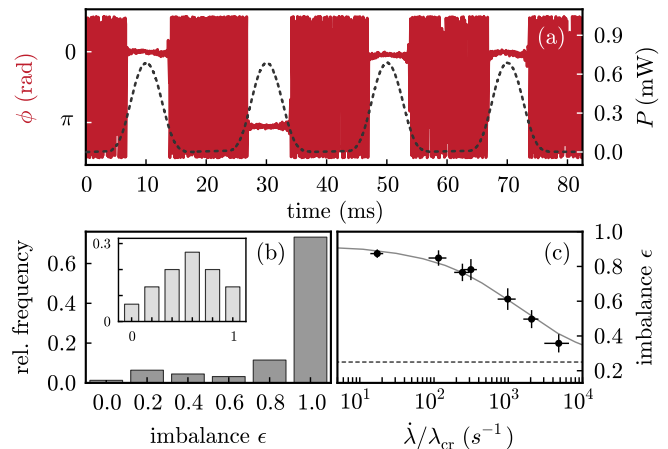


FIG. 3. (Color online) (a) Cavity time-phase (red, averaged over  $30 \mu\text{s}$ ) for a single run, and corresponding time sequence of the coupling laser power  $P$  (dashed). (b) Probability distribution of the imbalance  $\epsilon$  (see text) for 156 runs, where the phase transition was crossed at a rate of  $\dot{\lambda}/\lambda_{\text{cr}} = 18(3) \text{ s}^{-1}$ . The inset displays the distribution of post-selected data (see text). (c) Mean imbalance (dots) as a function of the rate  $\dot{\lambda}/\lambda_{\text{cr}}$  at which the transition was crossed (extracted from 356 runs in total), and theoretical model (solid line). The error bars indicate the standard error of the mean of  $\epsilon$  and systematic changes of  $\lambda_{\text{cr}}$  during probing.

In the experiment we determined the even-odd imbalance  $\epsilon$  for increasingly larger rates  $\dot{\lambda}/\lambda_{\text{cr}}$  at which the critical point was crossed, i.e. in an increasingly non-adiabatic situation (Fig. 3c). As the transition is crossed faster, the mean imbalance between the two superradiant phases decreases significantly and approaches the value  $\epsilon \approx 0.25$  corresponding to the balanced situation (Fig. 3c, dashed line). This indicates that the effect of the symmetry breaking term can be overcome by non-adiabatically crossing the phase transition.

Our observations (Fig. 3c) are in quantitative agreement with a simple model based on the adiabaticity condition known from the Kibble-Zurek theory [18, 19]. We divide the evolution of the system during the increase of the transverse laser power into a quasi-adiabatic regime, where the system follows the change of the control parameter, and an impulse regime, where the system is effectively frozen. After crossing the critical point, fluctuations of the order parameter, which are present at the instance of freezing, become instable and are amplified. The coupling strength which separates the two regimes is determined by Zurek's equation [18]  $|\dot{\zeta}/\zeta| = \Delta/\hbar$ , with  $\zeta = (\lambda_{\text{cr}} - \lambda)/\lambda_{\text{cr}}$  and the energy gap between ground and first excited state given by  $\Delta = \hbar\omega_0\sqrt{1 - \lambda^2/\lambda_{\text{cr}}^2}$  for  $\omega \gg \omega_0$  [13, 17].

We deduce the probability with which the system chooses the even phase,  $p_{\text{even}} = \int_0^\infty p(\Theta)d\Theta$ , from the probability distribution  $p(\Theta)$  at the instance of freezing, where  $\hat{\Theta}$  denotes the shifted dipole operator  $\hat{\Theta} = \hat{J}_x + \mathcal{O}$ .

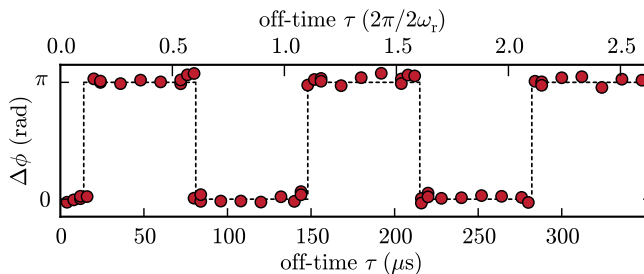


FIG. 4. (Color online) Coherent switching between the two ordered phases. After adiabatically preparing the system in one of the two superradiant phases, the coupling field is turned off for a time  $\tau$ . Displayed is the steady-state cavity time-phase  $\Delta\phi$  (averaged over 0.5 ms) after turning on the coupling field, referenced to the value recorded before the turning-off. Each data point corresponds to a single measurement. The dashed line shows the time evolution as expected from the two-mode model.

In the thermodynamic limit the distribution  $p(\Theta)$  becomes Gaussian with a mean value  $\langle\hat{\Theta}\rangle = \langle\hat{J}_x\rangle + \mathcal{O}$  and a width determined by the quantum fluctuations of the order parameter  $\Delta J_x$ . These values are determined from the linear quantum Langevin equations based on the Dicke model [17] including the symmetry breaking term. Besides the decay of the cavity field we also take into account dissipation of the excited momentum state at a rate  $\gamma = 2\pi \times 0.6$  kHz. This value was deduced from independent measurements of the cavity output field below threshold [20].

From the steady-state solution of the quantum Langevin equations we find that the mean order parameter  $\langle\hat{\Theta}\rangle$  grows faster in  $\lambda$  than its fluctuations. If  $\mathcal{O} > 12$  the order parameter exceeds its uncertainty already below critical coupling. Thermal fluctuations are neglected in this analysis. For our typical condensate temperatures of about 100 nK quantum fluctuations dominate as long as  $\zeta > 0.005$ . We account for shot-to-shot fluctuations of the overlap  $\mathcal{O}$  by suitably averaging over the position of the harmonic trap. The solid line in Fig. 3c shows a least square fit of our model to the data with the single free parameter  $\mathcal{O}_0$ . We obtain a value of  $\mathcal{O}_0 = 77$  which is in reasonable agreement with the theoretically expected value of  $\mathcal{O}_0 = 40$ . This verifies the predominance of the considered symmetry breaking field over other possible noise terms.

Finally, we experimentally demonstrate coherent switching between the two ordered states. To this end we suddenly turn off the coupling laser field after adiabatically preparing the system in one of the two superradiant phases. The atoms are then allowed to freely evolve according to their momentum state occupation, giving rise to standing-wave oscillations of the atomic density distribution. In the two-mode description this corresponds to harmonic oscillations of the order parameter  $\langle\hat{J}_x\rangle$  at frequency  $2\omega_r$ . We probe this time evolution by turning

on the coupling laser after a variable off-time  $\tau$ , thereby deterministically re-trapping the atoms either in the initial or in the opposite superradiant state. As expected, we observe regular  $\pi$ -jumps in the difference  $\Delta\phi$  between the steady-state phase signals measured before and after the free evolution, with a frequency of  $2\omega_r$  (Fig. 4, dashed line). The inertia of the atoms traveling at finite momentum causes the  $\pi$ -jumps in Fig. 4 to occur before those times at which the order parameter has evolved by an odd number of quarter periods.

In conclusion, we have experimentally monitored symmetry breaking in the Dicke quantum phase transition and identified the interplay between a residual symmetry breaking field, fluctuations and the crossing speed.

We acknowledge discussions with G. Blatter, P. Domokos, T. Donner, R. Landig, B. Oztot, L. Pollet, H. Ritsch, and H. Türeci. Financial funding from NAME-QUAM (European Union), SQMS (ERC) and QSIT (NCCR) is acknowledged.

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